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# Interactions of Axisymmetric and Non-Axisymmetric Disturbances in the Flow between Concentric Rotating Cylinders: Bifurcations near Multiple Eigenvalues<sup>†</sup>

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We consider the instability of Couette flow between rotating concentric cylinders of infinite length. Let  $R_1$ ,  $R_2$  and  $\Omega_1$ ,  $\Omega_2$  denote the radii and angular velocities of the inner and outer cylinders, respectively; and let  $\eta = R_1/R_2$  and  $\nu = \Omega_2/\Omega_1$ . We also introduce the Taylor number  $T = -4A\Omega_1 d^4/\nu^2$  with  $A = (\Omega_2 R_2^2 - \Omega_1 R_1^2)/(R_2^2 - R_1^2)$ .

It is known from the linear stability analysis of Krueger, Gross, and DiPrima (1966) for  $\eta$  near one that (i) for  $\nu = 0$  and increasing  $T$ , Couette flow first becomes unstable to an axisymmetric disturbance at  $T = T_c(\nu = 0, \eta)$ , (ii) in the absence of an axisymmetric disturbance Couette flow would become unstable to a non-axisymmetric disturbance at a value of  $T$  just slightly greater than  $T_c$ , and (iii) at  $\nu$  sufficiently negative the critical value of  $T$  occurs for a non-axisymmetric disturbance. Thus, for fixed  $\eta$  near one there are parameter values  $(\nu_1, T_1)$  for which axisymmetric and non-axisymmetric disturbances are simultaneously critical for Couette flow. We wish to study the bifurcation problem in the neighborhood of  $(\nu_1, T_1)$ , where we note that instability to an axisymmetric disturbance gives a steady bifurcation while instability to a non-axisymmetric disturbance gives a Hopf bifurcation.

In order to develop a mathematically tractable problem, we assume that the axial wavenumber of the axisymmetric and non-axisymmetric disturbances are the same, and we choose the wavenumber to be that corresponding to critical conditions for an axisymmetric disturbance. We let  $\lambda$  denote the dimensionless axial wavenumber (scaled with respect to  $R_2 - R_1$ ) and  $m$  denote the number of waves in the azimuthal direction. Dr. Peter Eagles has carried out the necessary calculations for  $\eta = 0.95$  to determine the points  $(\nu_1, T_1)$  for

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simultaneous instability to an axisymmetric disturbance  $(\lambda, 0)$  and a non-axisymmetric disturbance  $(\lambda, m)$  with the results given in the Table.

Table

$m$	$\mu$	$T_1$	$\lambda$
1	-0.73976	11,973.87	3.482
2	-0.75123	12,319.01	3.507
4	-0.80284	14,015.01	3.628

Thus, for example, for  $\eta = 0.95$  and  $\mu = -0.73976$ , Couette flow is simultaneously unstable to an axisymmetric disturbance with wavenumbers  $(3.482, 0)$  and a non-axisymmetric disturbance with wavenumbers  $(3.482, 1)$ . We also note that at  $(\mu_1, T_1)$  there are 6 critical modes with axial ( $Z$ ) and azimuthal ( $\theta$ ) dependence as follows:  $\cos \lambda Z$ ,  $\sin \lambda Z$ ,  $\exp(\pm i m \theta) \cos \lambda Z$  and  $\exp(\pm i m \theta) \sin \lambda Z$ . The first step in the bifurcation analysis is to represent the components of the disturbance by

$$\begin{aligned} & A_c u_c(r) \cos \lambda Z + A_s u_s(r) \sin \lambda Z + \\ & B_c v_c(r) \exp(i m \theta) \cos \lambda Z + B_s v_s(r) \exp(i m \theta) \sin \lambda Z + \\ & \bar{B}_c \bar{v}_c(r) \exp(-i m \theta) \cos \lambda Z + \bar{B}_s \bar{v}_s(r) \exp(-i m \theta) \sin \lambda Z + \Psi \end{aligned} \quad (1)$$

Here the functions  $u_c, u_s, v_c, v_s$  denote the radial dependence of the respective modes. The quantities  $A_c, A_s, B_c, B_s$  are scalar functions of time with  $A_c, A_s$  varying in the real numbers and  $B_c, B_s$  in the complex domain. The residual component  $\Psi$  is orthogonal to the six critical modes.

We are now able to reduce the nonlinear partial differential equation to a finite-dimensional center manifold of dimension six; this reduction is performed by expressing  $\Psi$  in terms of the coefficients  $A_c, A_s, B_c, B_s$ . The nonlinear ordinary differential equations for these coefficients describe the 'flow' in the center manifold and are similar to those derived by Davey, DiPrima and Stuart (1968) for the case  $\mu = 0$ .

We give particular attention to the interaction of the three modes  $\cos \lambda Z$  and  $\exp(\pm i m \theta) \sin \lambda Z$ , which can give rise to Taylor-vortex and wavy-vortex flows which are often observed in experiments. So we put  $A_s = B_c = 0$  and we obtain the following equations where terms of the order 7 have been omitted:

$$\begin{aligned}\frac{dA_c}{dt} &= a_0 A_c + a_1 A_c^3 + a_4 A_c |B_s|^2 + a_5 A_c^5 + a_6 A_c^3 |B_s|^2 + a_7 A_c |B_s|^4, \\ \frac{dB_s}{dt} &= b_0 B_s + b_1 B_s |B_s|^2 + b_4 B_s A_c^2 + b_5 B_s A_c^4 + b_6 B_s |B_s|^2 A_c^2 + b_7 B_s |B_s|^4.\end{aligned}\quad (2)$$

Complex conjugate equation for  $\bar{B}_s$ .

The scalar quantities  $a_j$  are real and the  $b_j$  are complex numbers. In the sequel we shall denote by  $b_{jr}$  and  $b_{ji}$  the real and imaginary parts of  $b_j$ . After performing the transformation  $A_c = x$ ,  $B_s = y \exp i(b_{0i}t + \phi)$  the equation for  $\phi$  uncouples from those for  $x$  and  $y$  and we obtain

$$\begin{aligned}\frac{dx}{dt} &= a_0 x + a_1 x^3 + a_4 xy^2 + a_5 x^5 + a_6 x^3 y^2 + a_7 xy^4, \\ \frac{dy}{dt} &= b_{0r} y + b_{1r} y^3 + b_{4r} x^2 y + b_{5r} x^4 y + b_{6r} x^2 y^3 + b_{7r} y^5.\end{aligned}\quad (3)$$

The coefficients  $a_j$  and  $b_{jr}$  depend on  $\nu$  and  $T$ , while  $a_0(\nu_1, T_1) = b_{0r}(\nu_1, T_1) = 0$ .

It is now possible to study changes in the phase plane as a function of the two parameters  $\nu - \nu_1$  and  $T - T_1$ . It is natural first to analyze system (3) with the fifth order terms left out. Keener (1976) and Langford and Iooss (1980) consider pairs of coupled ordinary differential equations with the same cubic nonlinearities as (3) by taking  $a_1, a_4, b_{1r}, b_{4r}$  fixed and varying  $a_0$  and  $b_{0r}$ . The nature of the changes in the phase plane depends of course on the specific values of the 3<sup>rd</sup> order coefficients and the cubic systems of o.d.e.'s can be classified accordingly.

While the investigations by Keener and Langford-Iooss were carried out for arbitrary nonvanishing values of the 3<sup>rd</sup> order coefficients, it turns out that in the problem which is presently under discussion some of these coefficients are very small and change sign for values of  $\nu, T$  very close to  $\nu_1, T_1$ . For example, if we let  $\nu$  take values close to  $\nu_1$  and compute  $T = T_c(\nu, \eta = .95, \lambda = 3.482, m = 0)$  then  $a_1(\nu, T) = 0$  for  $\nu = -.68939$ , which is indeed very close to  $\nu_1 = -.73976$ . For this reason the analysis of (3) has to be taken to fifth order and the dependence of  $a_1, a_4, b_{1r}, b_{4r}$  on  $\nu$  and  $T$  has to be taken into account up to first order in  $\nu - \nu_1$  and  $T - T_1$ . The numerical evaluation of the very complicated expressions for the fifth order coefficients have been carried out by Dr. Eagles and the analysis of the changes in the phase plane is in progress.

The full system of six amplitude equations is also studied, but only at cubic order in the amplitudes. For this case there does not appear to be a transformation that will reduce the order of the system.

For the case  $\nu = 0$ , we have noted that the axisymmetric and non-axisymmetric modes are not simultaneously unstable. However, it is possible to imbed this problem in a larger class of mathematical problems which we can analyze using the techniques of center manifold theory. In this way we present a rational basis (though admittedly complicated) for the analysis of the successive bifurcations from Couette flow to Taylor-vortex flow and then to wavy-vortex flow for increasing  $T$  with  $\nu = 0$  as first studied by Davey, DiPrima, and Stuart (1968).

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#### References

1. E.R. Krueger, A. Gross and R.C. DiPrima; 1966; On the relative importance of Taylor-vortex and non-axisymmetric modes in flow between rotating cylinders. *J. Fluid Mech.* 24, 521-538.
2. A. Davey, R.C. DiPrima, and J.T. Stuart; 1968; On the instability of Taylor vortices. *J. Fluid Mech.* 31, 17-52.
3. J.P. Keener; 1976; Secondary bifurcations in nonlinear diffusion reaction equations. *Studies in Appl. Math.* 55, 187-211.
4. W.F. Langford and G. Iooss; 1980; Interactions of Hopf and pitchfork bifurcations. To appear in *Birkhäuser Lecture Notes*, H.D. Mittelmann ed.